Differentiation: Standard Derivatives

In this worksheet we will first give a table of standard derivatives and then give a series of examples to show how the table is used. Note that in the table a will stand for a constant.

Some Common Derivatives

	$f(x)$	$f'(x)$	Comments
(1)	a	0	
(2)	x^n	nx^{n-1}	Here we must have $x \neq 0$ if $n < 1$
(3)	e^{ax}	ae^{ax}	
(4)	$\ln(ax)$	$\frac{1}{x}$	Here we must have $ax > 0$
(5)	$\sin(ax)$	$a\cos(ax)$	
(6)	$\cos(ax)$	$-a\sin(ax)$	Note the change of sign
(7)	$\tan(ax)$	$a\sec^2(ax)$	Here $ax \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

Example 1: Find the derivative of $f(x) = 5$.

Solution 1: Since 5 is a constant, we can use Rule (1) to obtain $f'(x) = 0$.

Example 2: Find the derivative of $f(x) = e^{\cos(2)}$.

Solution 2: Note that there is no x in $e^{\cos(2)}$, so it is still just a constant.
So we can can again use Rule (1) to obtain $f'(x) = 0$.

Example 3: Find the derivative of $f(x) = x^5$.

Solution 3: Here the function is of the form x^n with $n = 5$.
So we can use Rule (2) to obtain $f'(x) = 5x^{5-1} = 5x^4$.

Example 4: Find the derivative of $f(x) = x^{-3}$ if $x \neq 0$.

Solution 4: Here the function is of the form x^n with $n = -3$ and $x \neq 0$.
So we can again use Rule (2) to obtain $f'(x) = -3x^{-3-1} = -3x^{-4}$.

Example 4: Find the derivative of $f(x) = x^{\sin(3)}$ if $x \neq 0$.

Solution 4: Since $\sin(3)$ is just a number, the function is of the form x^n with $n = \sin(3)$ and $x \neq 0$.
So we can again use Rule (2) to obtain $f'(x) = \sin(3)x^{\sin(3)-1}$.

Example 5: Find the derivative of $f(x) = e^{2x}$.

Solution 5: The function is of the form e^{ax} with $a = 2$.
So we can use Rule (3) to obtain $f'(x) = 2e^{2x}$.

Example 6: Find the derivative of $f(x) = e^{-x}$.

Solution 6: The function is of the form e^{ax} with $a = -1$.
So we can again use Rule (3) to obtain $f'(x) = -1e^{-x} = -e^{-x}$.

Example 7: Find the derivative of $f(x) = \ln(8x)$ where $x > 0$.

Solution 7: The function is of the form $\ln(ax)$ with $a = 8$ and $ax > 0$.
So we can use Rule (4) to obtain $f'(x) = \dfrac{1}{x}$.

Example 8: Find the derivative of $f(x) = \ln\left(-\dfrac{7}{2}x\right)$ where $x < 0$.

Solution 8: The function is of the form $\ln(ax)$ with $a = -\dfrac{7}{2}$ and $ax > 0$.
So we can again use Rule (4) to obtain $f'(x) = \dfrac{1}{x}$.

Example 9: Find the derivative of $f(x) = \sin(6x)$.

Solution 9: The function is of the form $\sin(ax)$ with $a = 6$.
So we can use Rule (5) to obtain $f'(x) = 6\cos(6x)$.

Example 10: Find the derivative of $f(x) = \sin\left(-\dfrac{3}{2}x\right)$.

Solution 10: The function is of the form $\sin(ax)$ with $a = -\dfrac{3}{2}$.
So we can again use Rule (5) to obtain $f'(x) = -\dfrac{3}{2}\cos\left(-\dfrac{3}{2}x\right)$.

Example 11: Find the derivative of $f(x) = \cos(11x)$.

Solution 11: The function is of the form $\cos(ax)$ with $a = 11$.
So we can use Rule (6) to obtain $f'(x) = -11\sin(11x)$.

Example 12: Find the derivative of $f(x) = \cos(-\pi x)$.

Solution 12: The function is of the form $\cos(ax)$ with $a = -\pi$.
So we can again use Rule (6) to obtain $f'(x) = -(-\pi)\sin(-\pi x) = \pi\sin(-\pi x)$.

Example 13: Find the derivative of $f(x) = \tan(9x)$ where $9x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$.

Solution 13: The function is of the form $\tan(ax)$ with $a = 9$ and $ax \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$.
So we can use Rule (7) to obtain $f'(x) = 9\sec^2(9x)$.

Example 14: Find the derivative of $f(x) = \tan\left(-\dfrac{9}{8}x\right)$ where $-\dfrac{9}{8}x \neq \dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$.

Solution 14: The function is of the form $\tan(ax)$ with $a = -\dfrac{9}{8}$ and $-\dfrac{9}{8}x \neq \dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$.
So we can again use Rule (7) to obtain $f'(x) = -\dfrac{9}{8}\sec^2\left(-\dfrac{9}{8}x\right)$.